Econ 21410 - Problem Set VII

Linear Programming*

May 19, 2015

This homework should be done in LaTeX The homework will be graded on correctness, but will also heavily weight clarity and professionalism. Its better to not do some of the harder parts than to turn in an incomprehensible document. Your R script as well as a log-file should be submitted. Alternatively, use knitr to print your code inline in your latex document.

SUBMISSION: The homework must be emailed to Oliver and myself by 5p.m. Thursday, May 21th. The email must include a pdf with the filename lastname_pset6.pdf and R code called lastname_pset6_code.R where "lastname" should be replaced with your last name. The subject of your email should be [ECON 21410: pset7 submission]

If you are struggling, please use the github page to ask for help!* Remember that asking and answering questions on our github page, coming to office hours to ask questions, and contributing to the class wiki are all worth participation credit, which is 10% of your grade in this class.

1 Solving a Mixed Integer Linear Program

Consider the problem of a firm trying to satisfy customers $i = 1 \dots L$ each with a different demand distributed across Euclidean space. Each firm has a number of cantidate sites $j \in 1 \dots F$. Each site has a fixed cost f_j. Each site if built would have a maximum capacity of C units. There is a delivery cost from a build factory to a customer equal to the euclidian distance from that factory to the customer. Let y_j be 1 if candidate factory j is built and zero otherwise. Let $x_{i,j}$ be 1 if customer i receives goods from built candidate factory j. The firm is trying to solve the following problem:

$$\begin{array}{lcl} \min & \sum_{i\in L}\sum_{j\in F} c_{ij}x_{ij} + \sum_{j\in F} f_j y_j & \\ \text{s.t} & \sum_{j\in F} x_{ij} = 1 & \forall i \in L \\ & x_{ij} \leq y_j & \forall i \in L, j \in F \\ & \sum_{i\in L} d_i x_{ij} \leq C y_j & \forall j \in F \\ & x_{ij}, y_j \in \{0,1\} & \forall i \in L, j \in F \end{array}$$

1. Write an interpretation in words to each line of the optimization program written above.

*Please email johneric@uchicago.edu and obrowne@uchicago.edu if you have questions.

2. Rewrite the optimization problem above in standard linear program form:

$$\begin{aligned} \max \quad & c^T x & \\ \text{s.t} \quad & Ax & \leq b \\ & x & \geq 0 \end{aligned}$$

Here is code that generates the data (and loads the libraries you will need ot use to solve it)

```
# Section 0: Set Up
#========================

#Libraries
library(ggplot2)
library(lpSolve)
library(lpSolveAPI)

#Set Seed
set.seed(21410)

#Set Parameters
numCust = 50
numSites = 12
SiteCapacity = 35
MaxDemand = 10
xmax = 200
ymax = 100

#Define customer characteristics
Customers = data.frame(cbind(x = xmax * runif(numCust),
                             y = ymax * runif(numCust),
                             demand = MaxDemand * runif(numCust)))

#Define site characteristics
Sites     = data.frame(cbind(x = xmax * runif(numSites),
                             y = ymax * runif(numSites),
                             capacity = SiteCapacity))
Sites$fixed_cost <- (Sites$x-xmax/2)^2 + (Sites$y-ymax/2)^2

#========================
```

3. Calculate the distance between each customer and each candidate site.

4. How many decision variables are there in this problem? How many constraints are there in this problem? Use the LpSolveAPI command *make.lp* to create an LP with that number of variables and constraints

5. Create a vector of characters with the names of all of your decision variables. Then create a vector of numbers with all of the coefficents in your objective function. Define this objective using the command *set.objfn*

6. Using various loops (or otherwise) define all of your constraints, adding them using the command *add.constraint*

7. Use the *write.lp* command to write your LP to a file. Name this file appropriately and submit it with your pset.

8. Solve your LP using the command *solve*. Report the total cost. with the command *get.objective*. Then get the optimal value of the variables using the command *get.variables*. Report which factories are built. Report which factory each customer gets his supply from. [1]

9. To ensure you get binary solutions to y_i you need to use the command *set.type* to set the type of these variables to be "binary". However you do not need to put similar restrictions on the x_ij variables to get binary solutions. Can you explain why this is?

10. Rerun your simulation this time assuming that there is no fixed cost to building a plant. Again report the total cost. Report which factories are built. Report which factory each customer gets his supply from.

11. Using ggplot2 (or other plotting package) produce a plot of your results with the following features

    - Each of the customers as a transparent circle where the size of the circle is proportional to their demand
    - Each of the built factorys as a solid circle where the size of the circle is proportional to their capacity
    - Match the colors between the factorys and their customers in the optimal solution.
    - Put a black cross on the figure for each non-selected factory.
    - Draw line segments between each factory and its corresponding customers.

2 Setting up a linear program (OPTIONAL second problem for up to 3 side-project points)

Recall from class that the standard form of a linear program is written as follows:

$$\begin{aligned} \max \quad & c^T x & \\ \text{s.t} \quad & Ax & \leq b \\ & x & \geq 0 \end{aligned}$$

Formulate the following problem as a linear program in the standard form[2]:

[1] Make sure you generate the data using my code above with the same seed so we get the same results

[2] note you do not actually need to compute or solve this problem, just write out the set of equations that define it

Coal Management Problem

Suppose that a mining company is trying to minimize the cost of satisfing all of its contracts. it has $i = 1, \ldots, m$ mines, $j = 1, \ldots, J$ silos and $k = 1, \ldots, K$ customers. Let p_i be the production of mine i. let a_i and s_i denote the the ash and sulfur content of the coal produced by mine i in percentage terms. Any excess coal not shipped must be stored at the site of the mine at a storage cost of c_i^M per ton at mine i, and the mine has a maximum storage capacity of M_i

Let A_1 and C_1 be a $m \times J$ matricies where $a_{ij}^1 \in A_1$ is the maximum shipment capacity from mine i to silo j and $c_{ij}^1 \in C_1$ is the cost of shipping from mine i to silo j.

Let A_2 and C_2 be a $J \times K$ matricies where $a_{jk}^2 \in A_2$ is the maximum shipment capacity from silo j to customer k and $c_{jk}^2 \in C_2$ is the cost of shipping from mine i to silo j.

At the silo all of the coal is blended such that the sulfur and ash content of the coal shipped from the silo to a customer is the average of the sulfur and ash content of the coal the silo receives. Consumers each have an upper and lower bounds for the maximum amount of coal and ash they require $[u_k^a, l_k^a]$ and $[u_k^s, l_k^s]$. Further there additional is revenue earned of r_k^s for each percentage-point the ash content of coal delivered to customer k falls below their upper bound u_k^a.

Side Projects

- Write a function which solves LPs by implementing the revised simplex algorithm Oliver outlined in class (up to 5 points).
- Formulate the linear program in question 1 using either the AMPL or GAMS languages and then solve the problem by submitting it to the NEOS server: http://www.neos-server.org/neos/ (up to 3 points).
- Extend the problem in question 2 to the case where there are $t = 1 \ldots T$ time periods. Assume that mine production and demand vary across time. Further assume it takes one period to ship coal from the mine to the silo, one period to blend the coal and another period to ship the coal from the silo to the customer, so the coal arrives at the customer 3 periods after it leaves the mine. (up to 2 points).
- Write an outline up to one page discussing a problem that is frequently encountered in the economics literature can be reformulated as a linear program (up to 2 points).
- Write an outline up to one page reviewing the modern literature on Stigler's 1945 diet paper. What would such a diet contain today and how much would it cost? (up to 2 points)
- Write a 3-5 page literature review for your final research project (up to 3.5 points).
- Past side projects are still valid.