Econ 21410 - Problem Set III

Marriage Matching*

April 15, 2014

This homework should be done in LaTeX The homework will be graded on correctness, but will also heavily weight clarity and professionalism. Its better to not do some of the harder parts than to turn in an incomprehensible document. Your R script as well as a log-file should be submitted. Alternatively, use knitr to print your code inline in your latex document.

SUBMISSION: The homework must be emailed to Oliver and myself by 2p.m. Monday, April the 21st. The email must include a pdf with the filename lastname_pset2.pdf and R code called lastname_pset2_code.R where "lastname" should be replaced with your last name. The subject of your email should be [ECON 21410: pset3 submission]

If you are struggling, please use the github page to ask for help!* Remember that asking and answering questions on our github page, coming to office hours to ask questions, and contributing to the class wiki are all worth participation credit, which is 10% of your grade in this class.

Becker's Marriage Market Warm Up

1. Suppose the output of a marriage is determined by the function $h(m_i, f_j)$, where m_i and f_i are the amount of skill man i and woman j bring to the marriage respectively. For each of the functions below, answer if the function will lead to positive assortitive matching in (1) the transferable utility case and (2) the non-transferable utility case.

   - $h(m, f) = m^{0.3} f^{0.3}$
   - $h(m, f) = (m + f)^2$
   - $h(m, f) = (m + f)^{0.5}$
   - $h(m, f) = m + f$
   - $h(m, f) = min\{m, f\}$

*Please email johneric@uchicago.edu and obrowne@uchicago.edu if you have questions.

Becker's Marriage Market Warm up

1. Write out (in words) the steps for an algorithm that calculates the division of marital output in a marriage market with more women than men, men propose to women, and the output of the marriage is super-modular (so we have positive assortitive mating).[1]

2. Write out how this algorithm would change if there were more men than women, but men still proposed.

3. Assume that there are more fs than ms and that ms "propose" in this model.[2] Assume the utility of not marrying is 0. In class we showed that such a setup will have positive assortitive mating. Who will women i match with if i is less than the total number men?

4. Write a function that takes the "males" and "females" matrices defined below and calculates: (1) the output of each match (we already know from positive assortitive mating which male and female will match with each other) and (2) the division of the output between men and women. The function should fill in the columns of the "males" and "females" matrix and return those matrices in a list.

5. What proportion of the output do fs get when education has the discrete binomial distribution? Run the model a few times and make sure your initial run is not an outlier.

6. Change "males" and "females" to have education levels drawn from the uniform distribution (currently commented out in the code below). How does this change the proportion of the output that the fs get on average. Run the model a few times and make sure your initial run is not an outlier.

7. Discuss the differences between your results in the previous two questions. Explain the economics behind why they differ.

8. (if you are struggling with the problem set, skip the remaining two parts of this problem as they will be worth fewer points than the rest of the problem set.)

9. Extend your function to work in the case where there are more men than women, but men still propose.

10. What proportion of the output do fs now get when education is binomially distributed? How about when education is distributed uniformly? How does this differ from your result when there were fewer men than women.

```
# ====================== TITLE: computational economics: assignment 3

# AUTHOR: John Eric Humphries

# abstract: problem set on marriage matching

# Date: 2014-04-12 ====================
```

[1] Hint: We did this in class!

[2] This means men propose a division of the marriage output which women can accept or reject.

```r
# ======================== Section 0: setup ========================

# setwd('/mnt/ide0/home/johneric/sbox/projects/neighborhoodVision/')
rm(list = ls())  # Clear the workspace


# ================================ Section 1: Generating Agents with education for Becker
# Marriage model. ================================
nMales <- 100  # number of males
nFemales <- 110  # number of females

males <- matrix(0, nMales, 4)  # data for males to fill in
females <- matrix(0, nFemales, 4)  # data for females (some duplication)
colnames(males) <- c("id", "educ", "output", "surplus")
colnames(females) <- c("id", "educ", "output", "surplus")
males[, 2] <- rbinom(nMales, 16, 0.5)  # produces integer education (lots of overlap)
females[, 2] <- rbinom(nFemales, 16, 0.5)
# males[,2] <- runif(nMales,min=0,max=16) # produces decimal educations, much less overlap
# females[,2] <- runif(nFemales,min=0,max=16)

# Note that I am sorting the list in decreasing order of education.
males <- males[order(males[, 2], decreasing = T), ]
females <- females[order(females[, 2], decreasing = T), ]
males[, 1] <- c(1:nMales)
females[, 1] <- c(1:nFemales)
```

Gale Shapley

In the code below, I create a list of preference rankings for each man and woman in two different marriage markets. In "males1" and "females1", preferences over members of the opposite sex are random. In "males2" and "females2" there is assortitive matching where a lower id number is strictly preferred by everyone to a higher id number.

```r
# ======================== Section 3: Gale-Shapley ========================
nMales <- 10  # number of males
nFemales <- 10  # number of females
males1 <- matrix(replicate(nMales, sample(nFemales)), nMales, nFemales, byrow = T)
females1 <- matrix(replicate(nFemales, sample(nMales)), nFemales, nMales, byrow = T)

# Preferences from Becker's Marriage model (to see that we get the same results)
males2 <- matrix(replicate(nMales, c(1:nFemales)), nMales, nFemales, byrow = T)
females2 <- matrix(replicate(nFemales, c(1:nMales)), nFemales, nMales, byrow = T)
```

1. Write out the steps of the Gale-Shapley algorithm
2. Implement the Gale-Shapley algorithm. Write an algorithm that takes a matrix of men's rankings of women and a matrix of women's ranking of men as inputs. Have this function implement the Gale-Shapely algorithm and return the final "match matrix" MM which contains a 1 in cell $MM[i, j]$ if male i marries female j and contains a 0 otherwise.

3. Run the algorithm on the two populations below and print the output to screen. Make sure you get the correct result for the model with positive assortitive matching.

Research

In 1-3 sentences propose a research idea related to the marriage market.

Name two cases where matching models may be applicable outside of the marriage market.

Side Projects

- Side projects from previous homeworks are still valid (unless already completed, such as making a specific entry on the class wiki.)
- Read Prof Becker's original 1974 paper on this subject and write a short (1-3 page) summary and response (2.5 points).
- Rewrite some or all of the code above in Julia or C++ (with Rcpp) (up to 3.5 points).
- Read and review two applied papers which test or extend Becker's marriage model (1-3 pages, 2.5 points).
- Look for a paper which models how couples negotiate the division of output within a marriage and write a brief summary (1-2 pages, up to 2 points)
- Look for papers which discuss how divorce laws changed bargaining power. Read one and write a brief summary and response (1-2 pages, 2 points).
- Go to ipums.org and explore the variables in the latest wave of the American Community Survey (ACS) or the Consumer Population Survey (CPS). Find variables that are interesting or surprising and write up a 1-page report (up to 2 points).
- Have a research idea? Write a document that provides a (1) two sentence statement of the idea and (2) a more complete 0.3-1 page description of the idea (up to 3 points).